نفرة المعلومات والترميز - امتحان ثاني

Specialization:	Electrical Engineering		Palestinian National Authority Ministry Education & Higher Education Palestine Technical University College of Engineering & Technology
Course Name:	Information Theory and Coding		
Date:	14/04/2011		Second Exam Second semester 2010/2011
Time:	11:00-12:00		
Instructor:	Dr. Mutamed Khatib		
Name: [redacted] Answer key		Section:	/20

Answer *all* the following 4 questions

Q1. A $(7,4)$ block code has a parity check matrix:

$$\mathbf{H}=\begin{bmatrix}1 & 1 & 0 & 1 & 1 & 0 & 0\\ 1 & 1 & 1 & 0 & 0 & 1 & 0\\ 0 & 0 & 1 & 1 & 0 & 0 & 1\end{bmatrix}$$

6

This code can correct one error.

a. *(2 marks)* Derive the generator matrix of this code and encode the data 1110
b. *(2 marks)* Derive a syndrome decoding for the code as described above and decode the received data 1101110
~~c. *(2 marks)* Calculate the maximum number of errors a $(15,11)$ block code can correct.~~

$D_{min}=3$

Q2. *(10 marks)* For a $(6,3)$ systematic linear block code, the 3 parity check digits are:

$$P_1 = 1\times I_1 \oplus 1\times I_2 \oplus 1\times I_3$$
$$P_2 = 1\times I_1 \oplus 1\times I_2 \oplus 0\times I_3$$
$$P_3 = 0\times I_1 \oplus 1\times I_2 \oplus 1\times I_3$$

a. *(2 marks)* Construct the generator matrix for this code.
b. *(2 marks)* Construct all possible codewords generated by this matrix
c. *(2 marks)* Determine the error correcting capabilities for this code
d. *(2 marks)* Prepare a suitable decoding table
e. *(2 marks)* Decode the received words: 101100.

Q3. *(4 marks)* When generating a $(7,4)$ cyclic block code using the polynomial x^3+x^2+1:

a. *(2 marks)* What would the generated codeword be for the data sequence 1000?
b. *(1 marks)* Check that this codeword would produce a zero syndrome if received without errors.
c. *(1 marks)* If the codeword 1000110 is corrupted to 1001110, what is the syndrome at the receiver?

تم الرفع بواسطة
م. معن أبو عيسى

Good luck

Q.1

(a) $G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & & \\ 0 & 1 & 1 & & \\ 1 & 0 & 1 & & \end{bmatrix}$

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & \phi \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \quad (2)$$

$d = 1110 \Rightarrow c = dG = [1110010]$ (1)

(b)

0 0 0 0 0 0 0	0 0 0	
ϕ 0 0 0 0 0 0	ϕ 1 1	
0 1 0 0 0 0 0	ϕ 1 0	
0 0 1 0 0 0 0	0 1 ϕ	
0 0 0 1 0 0 0	1 0 1	(1)
0 0 0 0 1 0 0	ϕ 0 0	
0 0 0 0 0 1 0	0 1 0	
0 0 0 0 0 0 1	0 0 ϕ	

for $r = 1101110$

$$Hr = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ \phi & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} = [0\ 1\ 0]$$

$\Rightarrow$ error in bit # 6

$\Rightarrow d' = (1101100) \Rightarrow 1101$ (1)

(c) ~~$d_{min} = 3 \Rightarrow t = int\left(\frac{d_{min} - 1}{2}\right) = int\left(\frac{3-1}{2}\right) = 1$~~

(2)

Q.2

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$ (1)

(a) $\Rightarrow G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$ (2)

(b)

0 0 0	0 0 0 0 0 0
0 0 1	0 0 1 1 0 1
0 1 0	0 1 0 1 1 1
0 1 1	0 1 1 0 1 0
1 0 0	1 0 0 1 1 0
1 0 1	1 0 1 0 0 1
1 1 0	1 1 0 0 0 0
1 1 1	1 1 1 1 0 0

(2)

(2)

(c) $d_{min} = 3 \Rightarrow t = int\left(\frac{3-1}{2}\right) = 1$ error to be correct

(d)

0 0 0 0 0 0	0 0 0	
1 0 0 0 0 0	0 1 0	
0 1 0 0 0 0	1 1 1	(2)
0 0 1 0 0 0	0 1 1	
0 0 0 1 0 0	1 0 0	
0 0 0 0 1 0	1 0 1	
0 0 0 0 0 1	1 1 0	

(e) $Hr = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ (1)

$\Rightarrow$ error on the 2nd bit $\Rightarrow$ 1 1 1 1 0 0

(3)

Q.3 $g(x) = x^3 + x^2 + 1 \Rightarrow 1101$

(a)

1101 | 1000 000

1101

0101000

1101

011100

1101

00110 → 1000110

(2)

(b)

1101 | 1000110

1101

0101110

1101

011010

1101

000000 ⇒ ok! no errors.

(1)

(c)

1101 | 1001110

1101

0100110

1101

0100110

1101

(1)

(4)